

DEPENDENCE OF THE GLEISBERG CYCLE PERIOD ON THE LENGTH OF WOLF'S NUMBERS SERIES

Igor Shibaev¹, Aleksandr Shibaev²

¹*Pushkov Institute of Terrestrial Magnetism, Ionosphere and Radio Wave Propagation
Russian Academy of Sciences, Moscow, Russia.*

²*Lomonosov Moscow State University, Moscow, Russia
e-mail: ishib@izmiran.ru*

Keywords: “Gleisberg cycle”, Wolf numbers, Solar activity, Dalton minimum

Abstract

The concept of “Gleisberg cycle” arose from the analysis of a small amount of data for a series of Wolf numbers (WSN), which are characterized by varying degrees of reliability and with the key role of cycles 5–7 of the Dalton minimum. Back in the thirties of the last century, when analyzing the first 16 cycles was done, Gleisberg noted the frequency of their maximums in seven to eight cycles, and later gave an updated value of the period — about 80 years. In the works done over the past 60 years, this period is evaluated within 80–110 years. A number of researchers allocate a specific value for the Gleisberg cycle period equals to 88 years. Since different authors analyzed a series of Wolf numbers of different lengths, it makes sense to investigate the influence of the length of the series itself on this period.

The paper analyzes the long-period components of WSN versions v1 and v2. The connection between the period and the length of the series is found through the sine approximation of the corresponding fragments. An increase in the sine period from 82 to 110 years (for v1) was obtained with an increase in the length of the series from 18 to 24 cycles and the conditions for the local manifestation of the 88-year harmonic. The initial periodicity of the maximums of seven to eight cycles is transformed into ten to eleven cycles.

The WSN series includes recovered data from 1749 to 1849 and further on, regular observation data — reliable data. The dependence of the period on the length of the series, that is, on the share of reliable data, is associated with the inconsistency of the characteristics of the reconstructed and reliable series and casts doubt on the existence of the Gleisberg cycle or “secular” harmonic in the WSN readings in the 1749–2015 interval. Changes in the land cover by temporal analysis.

Introduction

The persistent interest in long-period solar activity (SA) cycles, including the Gleisberg cycle (which is often associated with the “secular” cycle), is associated with the manifestation of SA minimum / maximum epochs in everyday life. Back in 1939, Gleisberg, relying on the Zurich series (1750–1928) and smoothing the maximums of the cycles by four values, identified two maximums

and two minimums among them, which indicated the long-period component of the series (the Gleisberg cycle) with a frequency of their maximums/minimums of seven to eight cycles [1]. In later works [2, 3] a refined value of the period of about 80 years is given. Such an estimate can be easily obtained by approximating the amplitudes of these sixteen cycles with a sine, the parameters of which are found by the least squares method (OLS). The result, with the best value for the period of 82 years, is presented in Fig. 1, the dots mark the amplitudes themselves, along the horizontal axis of their date. Such an approximation satisfactorily describes no more than 18 cycles and its character is largely determined by cycles 5–7 of the Dalton minimum (MD).

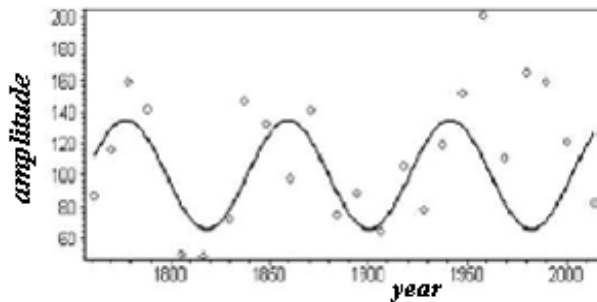


Fig. 1. Sine approximation of the amplitudes of cycles 1–16

In the works of various authors on the analysis of the WSN series itself, made over the past 60 years, the period of the Gleisberg cycle is estimated within 80–110 years. Researchers often identify a specific value for the period which is equal to 88 years [4–5]. Since different authors analyzed a series of different lengths, it makes sense to investigate the dependence of the period of the approximating sine on the length of the series itself, and besides this:

- To indicate the reasons for the growth (instability) of the period of the approximating sine.
- Point out the possible asymptotics of this growth.
- To specify more clearly such concepts as “period of the low-frequency component” and “epochs of minimum/maximum SA”.

Along with the classical version of the WSN series, its new version is also considered in the work. Since July 2015, the Belgian Center for the Study of the Sun (<http://sidc.oma.be>) has introduced new rules for calculating the monthly Wolf numbers, according to which the WSN series was recalculated from 1749 to June 2015. The ratio of the cycle maxima in the new v2 and old v1 versions of the WSN series is demonstrated by Fig. 2 [6]. Correction of a significant part of the old version of the series is formal, but the amplitude correction of cycles 10 and 18–24 will affect the “secular” component of the new version. In this work,

the presentation of the material and results are given for the old version of the series v1, the results associated with the new version v2 are marked or commented on.

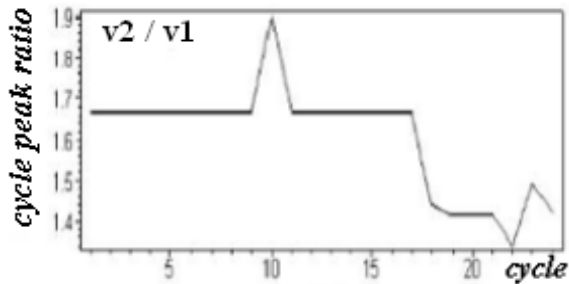


Fig. 2. The ratio of the maxima of the cycles

Initial data

Most of the results on the analysis of WSN were obtained before 2015, i.e., for version v1, and it is natural to rely on these developments. According to works [7–8], the long-period part of the v1 series, which includes components with a period of more than 24 years, is closely related to the envelope of the cycle maxima and serves as a geometrical place for the mean values of the cycles. This determines the choice of the source material, since Gleisberg analyzed the envelope of the cycle maxima. In this paper, we analyze the long-period components of the series of versions v1 and v2 (smoothed series of monthly Wolf numbers since 1749).

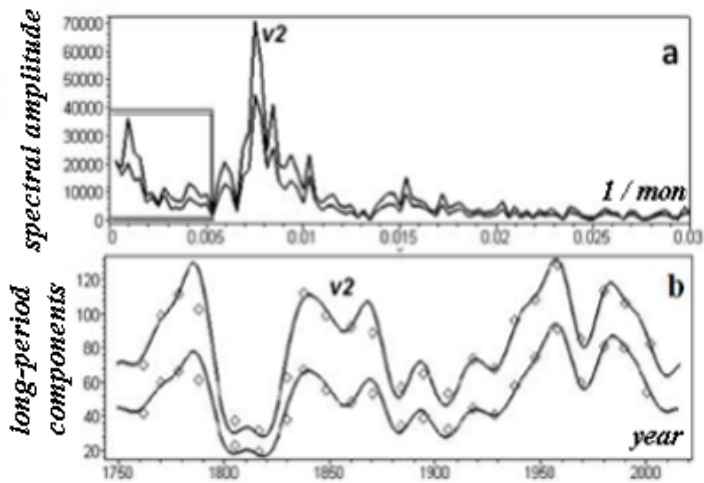


Fig. 3. Spectrums of WSN (a); long-period components (b)

The spectra of these series are presented in Fig. 3a, where the Ox axis is in reverse months, and the low-frequency part of the spectrum is labeled. The long-period components corresponding to the marked frequency range are displayed in Fig. 3b. The dots mark the mean values of the cycles which, for the v2 series [6], are well superimposed on the long-period components — the time axis Ox in years.

Sine-approximation of fragments of long-period components

Let us compare the approximations of two distinctive lengths of the low-frequency component of the old version of the series: a shortened fragment of eighteen cycles (1749–1954.37) and a long fragment up to the maximum of cycle 24 (1749–2014.376). The result of their approximation for $\sin((2\pi/T) + \varphi)$ is presented in Fig. 4. The period and phase of the sine were found by the least squares method, that is, by the minimum value of the sum squares of deviation (vertical axis) when scanning the corresponding rows with sine. The period was tested within 50–200 years (horizontal axis), phase $0 \dots 2\pi$. The series under study were preliminarily reduced to a commensurate scale — after subtracting the mean they were normalized to the square root of the variance. For the first row the smallest deviation is observed at a period of 84 years, for the second — at 110 years, the values of the phases at which the minima were reached are not indicated.

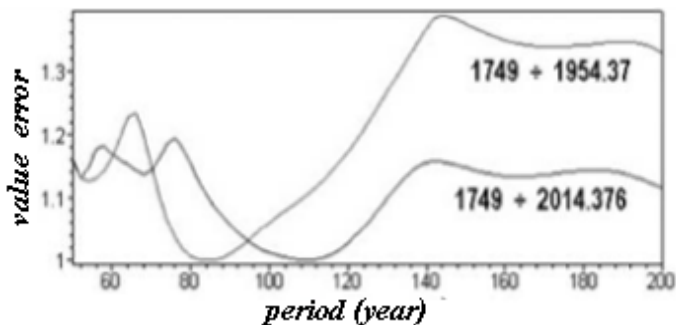


Fig. 4. Result of approximation of two options of long-period components by a sine

We use this approach to estimate the period of sine approximation of series of different lengths. The dependence of the period of the approximating sine (vertical axis in years) on the length of the series in cycles (the beginning of the cycle and its maximum) is shown in Fig. 5. The period grows with increasing row length and the 88-year harmonic appears in the old version of the series. The initial periodicity of the peaks of seven to eight cycles was transformed into ten to eleven cycles.

Results and Discussions

There are two important points to note:

- The above-mentioned estimate of the period of the Gleisberg cycle of 80–110 years (the results obtained by different authors at different times) coincided well with the ordered growth of the period from 82 to 110 years obtained in the work.
- The conditions for the local manifestation of the 88-year harmonic are clearly traced.

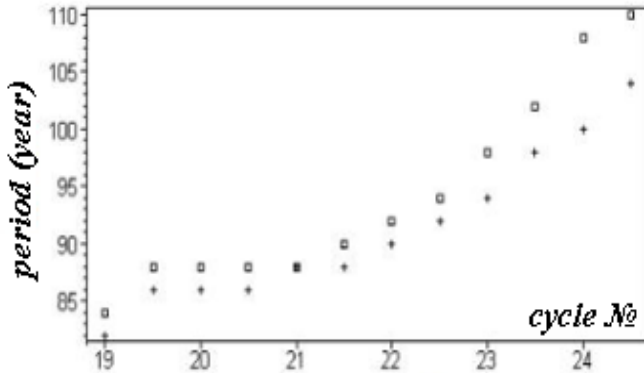


Fig. 5. Dependence of an approximating sine period on length of WSN; $v_2 - (+)$

It is clear that the instability (growth) of the estimate of the period of the “secular” harmonic complicates the interpretation and extrapolating it to the external time interval and identifying the SA minimum/maximum epochs. This behavior can be associated with an increase in the proportion of reliable data (cycles) in the analysis. Recall that the original series of monthly average Wolf numbers consists of the reconstructed series W_{rest} (from 1749 to 1849) and a reliable series of W_{tool} (regular instrumental observations from 1849 to the present), i.e. $W = W_{rest} \square W_{tool}$. Combining fragmentary data [9] with different density of observations, amplitude resolution and scaling will break the consistency of temporal fragments of different scales (for example, the structure of cycles and their relationship). All this manifested itself during the formation of the restored series. In the above mentioned works [7, 8] significant differences in the behavior of the series W_{rest} and W_{tool} are shown, the properties of the region of cycles 5–7 were significantly “distinguished”. With a series length of 18–19 cycles, a certain balance of properties of the restored and reliable parts is still preserved, and the Dalton minimum determines the formation and local manifestation of the 88-year to 90-year harmonic. A further increase in the proportion of reliable data shifts this balance in favor of the W_{tool} series with a smoother and more ordered long-period

component (Fig. 3b), which leads to an increase in the period. When analyzing only the W_{tool} series, the period of the approximating sine is 150 years [10], this corresponds to the periodicity of the maxima of fourteen cycles, and we have, as it were, the saturation of the period with the leveling of the role of the W_{rest} series, which corresponds to the concept under consideration. We simply state the same period for the new version W_{tool} equal to 131 years [6], taking into account the formal nature of the transformation of a significant group of cycles. Strictly speaking, the inconsistency of the parameters of the reliable and reconstructed series casts doubt on the existence of the Gleisberg cycle or “secular” harmonic in the WSN readings in the interval 1749–2015.

A different situation arises if one relies on the “good” data of the W_{tool} series to reconstruct the “bad” data of the W_{rest} series. Then we get “consistent” (probably without the Dalton minimum) behavior of the long-period part over the entire interval of 1749–2015 and an “adjusted” estimate of the average values of the cycles of the reconstructed series. This scenario concretizes concepts such as the “period of the low-frequency component” and “epochs of minimum/maximum SA”. An example of such an extrapolation, with a period of the approximating sine of the W_{tool} series of 150 and 131 years, is demonstrated in Fig. 6.

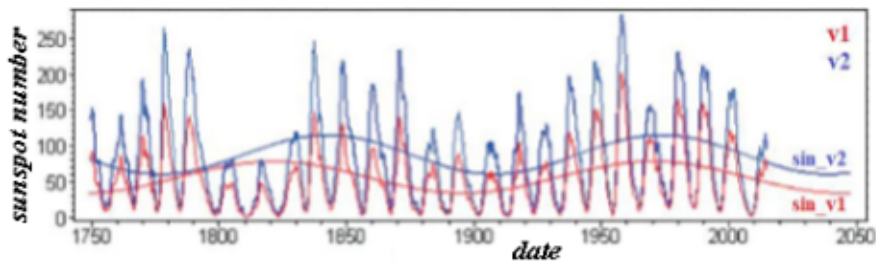


Fig. 6. Review of series $v1$, $v2$ and extrapolation of their sine approximations

Conclusion

As noted above, the concept of “Gleisberg cycle” arose from the analysis of small amount of data for a series of Wolf numbers of different reliability and with the key role of cycles 5–7. This fragment, with a low SA, coincided with a period of lower than average global temperatures. This justifies the presence of the Dalton minimum although such a connection is not obvious. Volcanic activity and elevated CO_2 levels may have a greater impact on climate than SA change [11]. Weather anomalies (“year without summer”) in Europe and America in 1816, caused by the eruption a year earlier of the Tambora volcano on the Indonesian peninsula Sumbawa, are the confirmation of this [12]. The temperature and SA in the recent past correlate in a different way: the temperature of the Earth has noticeably increased against the background of a rapid decrease in average activity and, since about 1970, the influence of the Sun on the climate could not be

significant [13]. With such a variety of situations, the plot presented in Fig. 6 is quite real. It should be expected that the agreement of the parameters of the reliable and reconstructed series will correct, first of all, the Dalton minimum, towards an increase in its values, and will change (cancel) the concept of the “Gleissberg cycle”. It is reasonable not to speak about the “Gleissberg cycle” tied to the MD, but about the long-period component corresponding to the W_{tool} series and the corrected W_{rest} series. Note that a critical attitude to the restored series is expressed by many authors in the works of the 1978 symposium — [14]. An attempt to balance the timing characteristics of the cycles of the same series at the expense of the “lost” cycle was undertaken in the work [15]. When analyzing the fractal properties of the series of the annual ring widths of eleven sequoias [16], the Dalton minimum did not appear.

Finally, we note that the closeness of the amplitude characteristics of cycles 8 and 9 to the parameters of reliable cycles allows us to speak about the consistency of the 150-year harmonic (generated by cycles 10–24) and the WSN readings since 1835.

The authors would like to sincerely thank V. N. Ishkov for the attention and support of the theme.

References

1. Gleissberg, W. A long-periodic Fluctuation of the Sun-spot Numbers. *Observatory*, 1939, 62, 158–159.
2. Gleissberg, W. Die Persistenz des 80 Jahrigen Sonnenfleckenzyklus Durch 16 Jahrhunderte. *Í. Ū. Fen Fak. Mecm. Seri C.*, 1955, 20, 263–271.
3. Gleissberg, W. Secularly Smoothed Data on the Minima and Maxima of Sunspot Frequency. *Solar Physics*, 1967, 2, 231–233.
4. Feynman, J., S. B. Gabriel. Period and phase of the 88-year Solar cycle and the Maunder minimum: evidence for a chaotic Sun. *Solar Physics*, 1990, 127, 2, 393–403.
5. Peristykh, A. N., P. E. Damon. Persistence of the Gleissberg 88-year solar cycle over the last ~12,000 years: Evidence from cosmogenic isotopes. *JGR*, 2003, 108, A1, 1003–1005. DOI:10.1029/2002JA009390.
6. Shibaev, A. The characteristics of old and new versions of month Wolf numbers range and there uniformity are compared. *Proceedings of 12th Scientific Conference with International Participation “Space, Ecology, Safety” SES’2016*, Sofia, Bulgaria, 2017, 73–76. (In Russian).
7. Shibaev I. G. Estimation of the reconstructed part of the Wolf series and the possibility of its correction. *Solar System Research*, 2008, 42, 1, 63–71.
8. Shibaev, I. G. Reconstruction of the Dalton Minimum using the 150-year harmonic and characteristics of reliable series of Wolf Numbers. *Proceedings of 12th Scientific Conference with International Participation “Space, Ecology, Safety - SES’2016”*, Sofia, Bulgaria, 2017, 68–72. (In Russian).
9. Hathaway, D. H. The Solar Cycle. *Living Rev. Solar Phys.*, 2015, 12, 4, 10–11. <https://doi.org/10.1007/lrsp-2015-4>.

10. Ishkov, V. N., I. G. Shibaev. Cycles of solar activity: general characteristics and modern boundaries of forecasting. *Izvestiya Rossiiskoi Akademii Nauk. Seriya Fizicheskaya*, 2006, 70, 10, 1439–1442.
11. Wagner, S., E. Zorita, The influence of volcanic, solar and CO₂ forcing on the temperatures in the Dalton Minimum (1790–1830): a model study. *Climate Dynamics*, 2005, 25, 205–218. DOI 10.1007/s00382-005-0029-0.
12. Schurer, A. P., G. C. Hegerl, J. Luterbacher *et al.* Disentangling the causes of the 1816 European year without a summer. *Environ. Res. Lett.*, 2019, 14, 11. <https://doi.org/10.1088/1748-9326/ab3a10>.
13. Krivova, N. A., S. K. Solanki. Solar variability and global warming: a statistical comparison since 1850. *Advances in Space Research*, 2004, 34, 361–364.
14. McCormac, B. *Solar-Terrestrial Influences on Weather and Climate. Proceeding of a Symposium*. Dordrecht, Holland, Springer, 1978, The Ohio State University, Columbus, Ohio, 24–28 August 1978, 348 p.
15. Usoskin, I. G., K. Mursula, G. A. Kovaltsov. The lost sunspot cycle: Reanalysis of sunspot statistics. *A&A*, 2003, 403, 2, 743–748.
16. Otsuka, K., G. Cornelissen, F. Halberg. Chronomics of tree rings gauge climate change. *Geophysical Processes and Biosphere*, 2009, 8, 1, 63–72. (In Russian).

ЗАВИСИМОСТЬ ПЕРИОДА ЦИКЛА ГЛЯЙСБЕРГА ОТ ДЛИНЫ РЯДА ЧИСЕЛ ВОЛЬФА

Иг. Шибеев, Ал. Шибеев

Резюме

Понятие «цикл Гляйсберга» возникло из анализа небольшого объема данных с различной степенью достоверности и с ключевой ролью минимума Дальтона. Сглаживая максимумы циклов Цюрихского ряда (1750–1928 г.) по четырем значениям Гляйсберг выделил среди них два максимума и два минимума, которые и указывали на длиннопериодную составляющую ряда с периодичностью максимумов/минимумов в семь–восемь циклов.

В работе анализируются длиннопериодные компоненты ряда WSN версий v1 и v2. Связь периода и длины ряда находится через синус-аппроксимацию соответствующих фрагментов. Для v1 получен рост периода синуса с 82 до 110 лет при увеличении длины ряда от 18 до 24 циклов, т. е. периодичность максимумов трансформируется в десять–одиннадцать циклов. Неустойчивая (растущая) оценка периода «вековой» гармонике затрудняет её интерпретацию и экстраполяцию на внешний временной интервал.

Ряд WSN включает восстановленные данные с 1749 по 1849 г. и далее данные регулярных наблюдений – достоверные данные. Зависимость периода от длины ряда, т. е. от доли достоверных данных, связана с несогласованностью характеристик восстановленного и достоверного рядов и ставит под сомнение существование «цикла Гляйсберга» или «вековой» гармонике в показаниях WSN на интервале 1749–2015 г.